

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Engineering Mathematics-I

Subject Code : 4TE01EMT1

Branch: B.Tech(All)

Semester : 1

Date :22/03/2017

Time :10:30 To 01:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q.1 Attempt the following questions: (14)

A) If the power of x & y both are even. then the curve is symmetrical about (1)

- (a) X-axis (b) Y-axis (c) both X & Y axes (d) none of these

B) If $y = \sin^{-1} x$, then x equal to (1)

- (a) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$ (b) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$
 (c) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$ (d) none of these

C) Define Euler's Theorem. (1)

D) If $x = r \cos \theta$ & $y = r \sin \theta$, then $\frac{\partial(x, y)}{\partial(r, \theta)} =$ _____ (1)

- (a) 0 (b) 1 (c) $\frac{1}{r}$ (d) r

E) $\lim_{x \rightarrow 0} \cos x =$ _____. (1)

- (a) 0 (b) 1 (c) ∞ (d) -1

F) $e^{10\pi i} =$ _____ (1)

- (a) 0 (b) 1 (c) -1 (d) None of these

G) Find $\frac{dy}{dx}$ for $x^2 + y^3 = 7xy$. (1)

H) If the two tangents at the point are real & distinct, the double point is called (1)

- (a) a node (b) a cusp (c) a conjugate point (d) None of these

I) The series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ represents expansion of (1)



- (a) $\sin x$ (b) $\cos x$ (c) $\cosh x$ (d) $\sinh x$

J) The series $\sum \frac{1}{n}$ is (1)

- (a) Convergent b) Divergent c) non-convergent d) a & b both

K) $\lim_{(x,y) \rightarrow (2,2)} \frac{3x+4y}{x+y} = \underline{\hspace{2cm}}$. (1)

- (a) 0 (b) 1 (c) $\frac{7}{2}$ (d) $\frac{3}{2}$

L) What is the argument of $z = 1 + i$ (1)

- (a) $-\frac{3\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

M) $\lim_{x \rightarrow 0} \frac{x - \sin x}{x} = \underline{\hspace{2cm}}$. (1)

- (a) 0 (b) 1 (c) ∞ d)-1

N) If $J = \frac{\partial(u,v)}{\partial(x,y)}$ & $J' = \frac{\partial(x,y)}{\partial(u,v)}$ then $JJ' = \dots\dots\dots$ (1)

- (a) 1 (b) -1 (c) 0 (d) None of these

Attempt any four questions from Q-2 to Q-8

Q.2 Attempt all questions (14)

A) Trace the curve (Cardioid) $r = a(1 + \cos \theta)$. (07)

B) Discuss the convergence of $\sum_{n=1}^{\infty} \left(\frac{\sqrt{n+1} - \sqrt{n}}{n} \right)$ (04)

C) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1^x + 2^x + 3^x}{3} \right)^{\frac{1}{x}}$. (03)

Q.3 Attempt all questions (14)

A) Test the Convergence of the series $\sum_{n=1}^{\infty} (ne^{-n^2})$. (05)

B) If $u = \ln(x^3 + y^3 + z^3 - 3xyz)$ then prove that (05)

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}$$

C) Show that $\frac{(\cos 2\theta - i \sin 2\theta)^5 (\cos 3\theta + i \sin 3\theta)^6}{(\cos 4\theta + i \sin 4\theta)^7 (\cos \theta - i \sin \theta)^8} = \cos 12\theta - i \sin 12\theta$. (04)

Q.4 Attempt all questions (14)

A) Test the series for absolute or conditional convergence (05)

$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots\dots$$



- B) Find the Taylor's expansion of $\tan\left(x + \frac{\pi}{4}\right)$ in ascending powers of x up to x^4 & find approximate value of $\tan(43^\circ)$. (05)
- C) Discuss the continuity of $f(x, y) = \begin{cases} \left(\frac{x^3 - y^3}{x^2 + y^2}\right); (x, y) \neq (0, 0) \\ 0; (x, y) = (0, 0) \end{cases}$ (04)

Q.5 Attempt all questions (14)

- A) Trace the curve (Cisoid of Diocle) $y^2(2a - x) = x^3$. (07)
- B) Find & plot all roots of $\sqrt[3]{8i}$. (04)
- C) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\tan^2 x}\right)$. (03)

Q.6 Attempt all questions (14)

- A) State Modified Euler's Theorem. If $u = \tan^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$, then (05)

show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$

- B) Prove that $\cos^8 \theta = \frac{1}{128} [\cos 8\theta + 8 \cos 6\theta + 28 \cos 4\theta + 56 \cos 2\theta + 35]$ (05)
- C) Define $\log(x + iy)$ and Show that the set of $\log(i^2)$ is not same as the set of values of $2 \log(i)$. (04)

Q.7 Attempt all questions (14)

- A) Find maxima & minima of the Function $f(x, y) = x^2 y - xy^2 + 4xy - 4x^2 - 4y^2$. (05)
- B) Obtain the range of convergence $\sum_{n=1}^{\infty} \left(\frac{x^n}{2^n}\right), x > 0$. (05)
- C) Find $\frac{dy}{dx}$ if $y^{x^y} = \sin x$ (04)

Q.8 Attempt all questions (14)

- A) Find the radius of convergence & interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$. (05)
- B) Define Errors. Find the percentage error in calculating the area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, when error of 1% is made in measuring its major & find minor axes. (05)
- C) If $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$, find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$. (04)

